$$\int \sqrt{16-t^2}\,dt$$

Para calcular esta integral es necesario hacer un cambio de variable diferente a lo que hemos visto hasta ahora. A continuación tienes el paso a paso:

$$\int \sqrt{16-t^2}\,dt = \left| t=4\,\text{sen}(x) \rightarrow dt=4\cos(x)\,dx \right| =$$

$$= \int \sqrt{16-16\,\text{sen}^2(x)}\cdot 4\cos(x)\,dx = \int 4\cos(x)\cdot 4\cos(x)\,dx = 16\int \mathbf{\cos^2(x)\,dx}$$

$$\int \cos^2(x)\,dx = \int \cos(x)\cdot\cos(x)\,dx = \left| \begin{array}{c} u=\cos(x)\rightarrow du=-\text{sen}(x)\,dx \\ dv=\cos(x)\,dx \rightarrow v=\text{sen}(x) \end{array} \right| =$$

$$= \text{sen}(x)\cdot\cos(x) + \int \text{sen}^2(x)\,dx = \text{sen}(x)\cdot\cos(x) + \int (1-\cos^2(x))\,dx =$$

$$= \text{sen}(x)\cdot\cos(x) + \int dx - \int \cos^2(x)\,dx = \text{sen}(x)\cdot\cos(x) + x - \int \cos^2(x)\,dx$$

$$\Rightarrow \int \cos^2(x)\,dx = \text{sen}(x)\cdot\cos(x) + x - \int \cos^2(x)\,dx$$

$$\Rightarrow 2\cdot\int \cos^2(x)\,dx = \text{sen}(x)\cdot\cos(x) + x$$

$$\Rightarrow \int \cos^2(x)\,dx = \frac{1}{2}\,\text{sen}(x)\cdot\cos(x) + \frac{x}{2}$$

$$16\int \cos^2(x)\,dx = 16\left(\frac{1}{2}\,\text{sen}(x)\cdot\cos(x) + \frac{x}{2}\right) = \left| \begin{array}{c} \text{sen}(x)=\frac{t}{4} \\ \cos(x)=\sqrt{1-\left(\frac{t}{4}\right)^2} \end{array} \right| = 8\,\frac{t}{4}\sqrt{1-\left(\frac{t}{4}\right)^2} + 8\,\text{arcsen}\left(\frac{t}{4}\right) =$$

$$= \frac{t}{2}\sqrt{16-t^2} + 8\,\text{arcsen}\left(\frac{t}{2}\right) + k$$