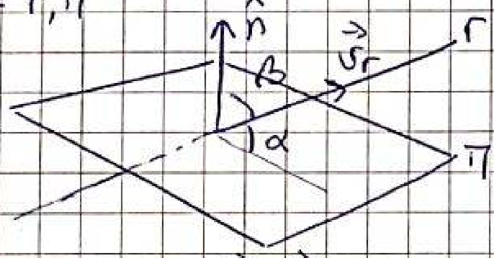
OPCIÓN A.

1. $r \equiv \frac{x+1}{2} = \frac{y-1}{-1} = \frac{z-2}{1}$; $\pi \equiv x - 2y - z = 4$

a) $\alpha = \widehat{r,\pi}$



$\vec{v_r}\,(2,-1,1)$; $|\vec{v_r}| = \sqrt{2^2+(-1)^2+1^2} = \sqrt{6}$

$\vec{n}\,(1,-2,-1)$; $|\vec{n}| = \sqrt{1^2+(-2)^2+(-1)^2} = \sqrt{6}$

$\vec{v_r}\cdot\vec{n} = |\vec{v_r}|\cdot|\vec{n}|\cdot\cos\beta = |\vec{v_r}|\cdot|\vec{n}|\cdot \operatorname{sen}\alpha$

$\Rightarrow \operatorname{sen}\alpha = \frac{\vec{v_r}\cdot\vec{n}}{|\vec{v_r}|\cdot|\vec{n}|} = \frac{2+2-1}{\sqrt{6}\cdot\sqrt{6}} = \frac{3}{6} = \frac{1}{2} \Rightarrow \boxed{\alpha = 30^\circ} = \frac{\pi}{6}\text{ rad}.$

b) $\pi' \perp \pi \Rightarrow \vec{n}$ es una dirección del plano π'

$r \in \pi' \Rightarrow \vec{v_r}$ es un dirección del plano π'

Cualquier punto de r es punto de $\pi' \Rightarrow P(-1,1,2) \in \pi'$

$$\boxed{\pi' = \begin{cases} x = -1 + \lambda + 2\mu \\ y = 1 - 2\lambda - \mu \\ z = 2 - \lambda + \mu \end{cases}}$$

$$\vec{n}\times\vec{v_r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & -1 \\ 2 & -1 & 1 \end{vmatrix} = (-3,-3,3)$$

$\pi' = -3x + 3y + 3z + D = 0$

$P \in \pi' \Rightarrow -3\cdot(-1) - 3\cdot 1 + 3\cdot 2 + D = 0$

$3 - 3 + 6 + D = 0 \Rightarrow D = -6$

$$\boxed{\pi' = -3x - 3y + 3z - 6 = 0}$$

$(x + y - z + 2 = 0)$

A.1

2. $r \equiv \begin{cases} x = 1+2a \\ y = a \\ z = 2-a \end{cases}$ $\qquad s \equiv \begin{cases} x = -1 \\ y = 1+b \\ z = -1-2b \end{cases}$ $\qquad a, b \in \mathbb{R}$

a) $P \in r, s$

$$\left.\begin{array}{r} 1+2a = -1 \\ a = 1+b \\ 2-a = -1-2b \end{array}\right\} \begin{array}{l} \to a = -1 \\ \longrightarrow b = -2 \end{array}$$

$$P \to \begin{cases} x = -1 \\ y = -1 \\ z = 3 \end{cases} \Rightarrow \boxed{P(-1,-1,3)}$$

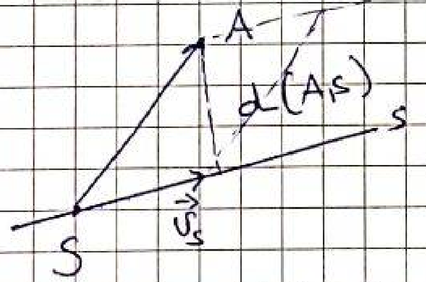
b) $\left.\begin{array}{l} \vec{v_r}\,(2,1,-1) \\ \vec{v_s}\,(0,1,-2) \end{array}\right\} \to$ direcciones del plano.

Como contiene a las dos rectas, tomamos un punto cualquiera de alguna de las rectas: $P(-1,-1,3)$

$$\boxed{\Pi \equiv \begin{cases} x = -1+2\lambda \\ y = -1+\lambda+\mu \\ z = 3-\lambda-2\mu \end{cases}} \qquad \Pi \equiv \begin{vmatrix} x+1 & y+1 & z-3 \\ 2 & 1 & -1 \\ 0 & 1 & -2 \end{vmatrix} = \underline{\underline{-x+4y+2z-3=0}}$$

c) $d(A,s)$

$A(0,0,1)$



$S(-1, 1+\cancel{b}, -1-\cancel{2b}) \in s$

$\vec{AS}\,(-1, 1+\cancel{b}, -2)$

$$d(A,s) = \frac{|\vec{AS} \times \vec{v_s}|}{|\vec{v_s}|} = \frac{\sqrt{1^2+2^2}}{\sqrt{1^2+(-2)^2}} = \underline{\underline{1\,u}}$$

$$\vec{AS} \times \vec{v_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & -2 \\ 0 & 1 & -2 \end{vmatrix} = (0,1,2)$$

A.2

8. Vectores ortogonales a $\vec{u}$ y $\vec{v}$: $\vec{u} \times \vec{v}$ y $\vec{v} \times \vec{u}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & -1 \\ -1 & 2 & 1 \end{vmatrix} = (1, 0, 1)$$

$$\vec{v} \times \vec{u} = -\vec{u} \times \vec{v} = (-1, 0, 1)$$

Para que su módulo sea 2, multiplicamos por: $\frac{2}{|\vec{u} \times \vec{v}|}$

$$|\vec{u} \times \vec{v}| = \sqrt{2}$$

$\Rightarrow \boxed{\vec{n}_1 (\sqrt{2}, 0, \sqrt{2}) \quad y \quad \vec{n}_1 (-\sqrt{2}, 0, -\sqrt{2})} \rightarrow$ vectores ortogonales de módulo 2.

4. $P_1(1,3,-1)$; $P_2(a,2,0)$; $P_3(1,5,4)$; $P_4(2,0,2)$

a) $\overrightarrow{P_1P_2}(a-1,-1,1)$
$\overrightarrow{P_1P_3}(0,2,5)$
$\overrightarrow{P_1P_4}(1,-3,3)$

Deben ser linealmente dependientes:

$$\begin{vmatrix} a-1 & -1 & 1 \\ 0 & 2 & 5 \\ 1 & -3 & 3 \end{vmatrix} = 6(a-1)-5-2+15(a-1)=0$$

$$21a-26=0 \Rightarrow \boxed{a=\frac{26}{21}}$$

b) Volumen de tetraedro $= \frac{1}{6}\left[\overrightarrow{P_1P_2}, \overrightarrow{P_1P_3}, \overrightarrow{P_1P_4}\right] = \frac{1}{6}(21a-26) = 7$

$$\Rightarrow 21a-26=42 \Rightarrow 21a=16 \Rightarrow \boxed{a=\frac{16}{21}}$$

c) Plano mediador de P_1 y P_3.

$P(x,y,z)$ es un punto del plano: $d(P,P_1)=d(P,P_3)$

$$\Rightarrow \sqrt{(x-1)^2+(y-3)^2+(z+1)^2} = \sqrt{(x-1)^2+(y-5)^2+(z-4)^2}$$

$$(x-1)^2+(y-3)^2+(z+1)^2 = (x-1)^2+(y-5)^2+(z-4)^2$$

$$x^2-2x+1+y^2-6y+9+z^2+2z+1 = x^2-2x+1+y^2-10y+25+z^2-8z+16$$

$$\boxed{4y-10z+31=0}$$

A.4

OPCIÓN B.

1) a) $\pi \equiv \begin{cases} x = 1+2a-b \\ y = -3+a \\ z = 2+3b \end{cases}$ $\longrightarrow a = y+3$, $\longrightarrow b = \dfrac{z-2}{3}$ $\longrightarrow x = 1+2(y+3) - \dfrac{z-2}{3} \Rightarrow$

$\Rightarrow 3x = 3 + 6(y+3) - z + 2$

$3x = 3 + 6y + 18 - z + 2$

$\boxed{\pi \equiv 3x - 6y + z = 23}$

— o —

$\vec{u}(2,1,0)$

$\vec{v}(-1,0,3)$ $\Rightarrow \vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 0 \\ -1 & 0 & 3 \end{vmatrix} = (3,-6,1)$

$P(1,-3,2)$ $\longrightarrow$ $3x - 6y + z + D = 0$

$3 \cdot 1 - 6(-3) + 2 + D = 0 \Rightarrow D = -23$ $\Big\}$ $\pi \equiv \underline{3x - 6y + z - 23 = 0}$

b) $r \perp \pi$

$P(-1,2,3) \in r$ $\Big\}$ $\vec{v}_r = \vec{n}(3,-6,1)$

$\boxed{r \equiv \begin{cases} x = -1 + 3\lambda \\ y = 2 - 6\lambda \\ z = 3 + \lambda \end{cases}}$

② $r \equiv \begin{cases} x-2y-2z=1 \\ x+5y-z=0 \end{cases}$ $\quad \pi \equiv 2x+y+nz=p$

a) r y π se cortan en un pto ⇒ el sistema $\begin{cases} r \\ \pi \end{cases}$ es S.C.D.

$$\left.\begin{array}{l} x-2y-2z=1 \\ x+5y-z=0 \\ 2x+y+nz=p \end{array}\right\} \quad A=\begin{pmatrix} 1 & -2 & -2 \\ 1 & 5 & -1 \\ 2 & 1 & n \end{pmatrix} \quad A'=\begin{pmatrix} 1 & -2 & -2 & 1 \\ 1 & 5 & -1 & 0 \\ 2 & 1 & n & p \end{pmatrix}$$

SCD ⇒ rango (A) = rango (A') = 3

$$|A| = 5n-2+4+20+2n+1 \neq 0$$

$$7n \neq -23 \Rightarrow \boxed{\forall n \neq -\frac{23}{7}}$$ r y π se cortan en un pto.

b) $r \in \pi$ ⇒ SCI con rango (A) = rango (A') = 2

$$|A| = 0 \Rightarrow n = -\frac{23}{7}$$

$$\begin{vmatrix} 1 & -2 & 1 \\ 1 & 5 & 0 \\ 2 & 1 & p \end{vmatrix} = 0 \Rightarrow 5p+1-10+2p=0 \Rightarrow 7p=9 \Rightarrow p=\frac{9}{7}$$

$$\begin{vmatrix} 1 & -2 & 1 \\ 1 & -1 & 0 \\ 2 & -\frac{23}{7} & p \end{vmatrix} = 0 \Rightarrow -p-\frac{23}{7}+2+2p=0 \Rightarrow p=\frac{9}{7}$$

$$\begin{vmatrix} -2 & -2 & 1 \\ 5 & -1 & 0 \\ 1 & -\frac{23}{7} & p \end{vmatrix} = 0 \Rightarrow 2p-\frac{115}{7}+1+10p=0 \Rightarrow 12p=\frac{108}{7} \Rightarrow p=\frac{9}{7}$$

$$\Rightarrow \boxed{n=-\frac{23}{7} \text{ y } p=\frac{9}{7}}$$

c) $r \parallel \pi$ ⇒ SI con rango $(A)=2$ y rango $(A')=3$

$$|A| = 0 \Rightarrow \boxed{n=-\frac{23}{7} \quad \text{y} \quad p \neq \frac{9}{7}}$$

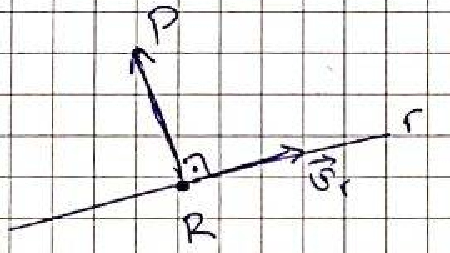
B.2

③ $P(3,-1,2)$

$d(P,r)$?

$$r \equiv \begin{cases} x - y + z = 1 \\ x + z = 0 \end{cases}$$

$$z = \lambda$$
$$x = -\lambda \quad \rightarrow \quad r \equiv \begin{cases} x = -\lambda \\ y = -1 \\ z = \lambda \end{cases} \qquad R(-\lambda,-1,\lambda) \in r$$
$$y = -1 \qquad\qquad\qquad\qquad\qquad \vec{v_r}(-1,0,1)$$



$$\vec{RP}\,(3+\lambda,\,0,\,2-\lambda)$$
$$\vec{RP}\cdot\vec{v_r} = -3-\lambda+2-\lambda = 0 \Rightarrow \lambda = \frac{-1}{2}$$
$$\rightarrow R\left(\frac{1}{2},-1,\frac{-1}{2}\right)$$

$$d(r,P) = d(R,P) = \sqrt{\left(3+\frac{1}{2}\right)^2 + \left(2+\frac{1}{2}\right)^2} = \frac{5}{\sqrt{2}}\,u$$

B.3

④ $r \equiv \frac{x-2}{3} = \frac{y-1}{-5} = \frac{z}{2}$

$s \equiv \begin{cases} x = 1-a \\ y = 3+a \\ z = 5 \end{cases}$

a) Posición relativa: $r \equiv \begin{cases} x = 2+3\lambda \\ y = 1-5\lambda \\ z = 2\lambda \end{cases}$

$\left.\begin{array}{l} 1-a = 2+3\lambda \\ 3+a = 1-5\lambda \\ 5 = 2\lambda \end{array}\right\} \rightarrow \left.\begin{array}{l} 3\lambda + a = -1 \\ -5\lambda - a = 2 \\ 2\lambda = 5 \end{array}\right|$

$A = \begin{pmatrix} 3 & 1 \\ -5 & -1 \\ 2 & 0 \end{pmatrix}$; $A' = \begin{pmatrix} 3 & 1 & -1 \\ -5 & -1 & 2 \\ 2 & 0 & 5 \end{pmatrix}$

$|A'| = -15 + 4 - 2 + 25 = 12 \neq 0 \Rightarrow$ rango$(A') = 3$
rango$(A) = 2$ } S. I.

Tienen distintas direcciones.

$\Rightarrow$ $\boxed{\text{r y s se cruzan.}}$

b) $S(1-a, 3+a, 5) \in s$
$R(2+3\lambda, 1-5\lambda, 2\lambda) \in r$ } $\vec{RS}(-1-a-3\lambda, 2+a+5\lambda, 5-2\lambda)$

$\vec{RS} \perp \vec{v}_r(3,-5,2) \Rightarrow \vec{RS} \cdot \vec{v}_r = 0 \Rightarrow -3-3a-9\lambda-10-5a-25\lambda+10-4\lambda = 0$

$-3-8a-38\lambda = 0$

$\vec{RS} \perp \vec{v}_s(-1,1,0) \Rightarrow \vec{RS} \cdot \vec{v}_s = 0 \Rightarrow 1+a+3\lambda+2+a+5\lambda = 0$

$3+2a+8\lambda = 0$

$\left.\begin{array}{l} -3-8a-38\lambda = 0 \\ 3+2a+8\lambda = 0 \end{array}\right\} \rightarrow \begin{array}{l} -3-8a-38\lambda = 0 \\ 12+8a+32\lambda = 0 \\ \hline 9 \quad\quad -6\lambda = 0 \end{array} \Rightarrow \lambda = \frac{3}{2} \Rightarrow$

$3+2a+12 = 0 \Rightarrow a = -\frac{15}{2}$

B.4

$d(r,s) = |\vec{RS}| = \sqrt{\left(-1+\frac{15}{2}-\frac{9}{2}\right)^2 + \left(2-\frac{15}{2}+\frac{15}{2}\right)^2 + (5-3)^2} = \sqrt{2^2+2^2+2^2} = \boxed{2\sqrt{3}\,u}$