Bloques Análisis y Álgebra.

Opción A

① $f(x) = \dfrac{e^x}{x-1}$; $x \neq 1$

a) Asintotas:

A.V. : $\lim_{x \to 1^-} \dfrac{e^x}{x-1} = \dfrac{e}{0^-} = -\infty$
$\lim_{x \to 1^+} \dfrac{e^x}{x-1} = \dfrac{e}{0^+} = +\infty$ } Asintota vertical en $x=1$.

A.H. : $\lim_{x \to +\infty} \dfrac{e^x}{x-1} = \dfrac{\infty}{\infty}$ → Aplicamos L'Hôpital : $\lim_{x \to +\infty} \dfrac{e^x}{1} = +\infty$

$\lim_{x \to -\infty} \dfrac{e^x}{x-1} = \dfrac{0}{\infty} = 0$ ⇒ Asintota horizontal en $y=0$, sólo cuando $x \to -\infty$

A.O. : sólo la estudiamos en $x \to +\infty$.

$\lim_{x \to +\infty} \dfrac{e^x/x-1}{x} = \dfrac{e^x}{x^2-x} = +\infty$ ⇒ No hay asintota oblicua.

b) Crecimiento y decrecimiento:

$f'(x) = \dfrac{e^x(x-1) - e^x}{(x-1)^2} = 0 \Rightarrow e^x(x-2) = 0 \Rightarrow x = 2$

$x < 1$: $f'(x) < 0$ — La función es decreciente en $(-\infty, 1) \cup (1, 2)$

$1 < x < 2$: $f'(x) < 0$ — La función es creciente en $(2, +\infty)$

$x > 2$: $f'(x) > 0$ — Tiene un mínimo en $(2, e^2)$.

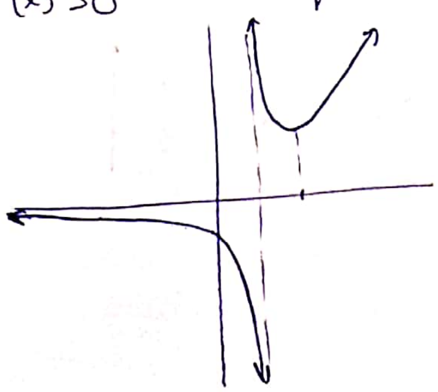
c) Concavidad y convexidad:

$f''(x) = \dfrac{e^x(x^2 - 4x + 5)}{(x-1)^3} = 0 \Rightarrow x^2 - 4x + 5 = 0$ → no tiene solución.

$x < 1$: $f''(x) < 0$ — La función es cóncava en $(-\infty, 1)$

$x > 1$: $f''(x) > 0$ — La función es convexa en $(1, +\infty)$

d)



② $\displaystyle\int \frac{3x^3+x^2-10x+1}{x^2-x-2}\,dx = \int\left(3x+4+\frac{9}{x^2-x-2}\right)dx = \int\left(3x+4+\frac{3}{x-2}-\frac{3}{x+1}\right)dx$

$$\begin{array}{l|l} 3x^3+x^2-10x+1 & x^2-x-2 \\ \hline 4x^2-4x+1 & 3x+4 \\ 9 & \end{array} \qquad \frac{9}{(x-2)(x+1)} = \frac{3}{x-2} - \frac{3}{x+1}$$

$$\rightarrow \int\left(3x+4+\frac{3}{x-2}-\frac{3}{x+1}\right)dx = \boxed{\frac{3}{2}x^2+4x+3\ln|x-2|-3\ln|x+1|+\text{Cte.}}$$

③ $f(x)=\begin{cases} \sqrt{ax} & ;\ 0\le x\le 8 \\ \dfrac{x^2-32}{x-4} & ;\ x>8 \end{cases}$ $\rightarrow$ continua.

a) $\sqrt{ax}$ es continua en $[0,8]$ si $a\ge 0$.

Si $a<0 \Rightarrow$ la función no existe en ese intervalo.

$\dfrac{x^2-32}{x-4}$ es continua en $(8,+\infty)$.

Estudiamos $x=8$:

$$\left.\begin{array}{l} \lim\limits_{x\to 8^+} f(x) = 8 \\ \lim\limits_{x\to 8^-} f(x) = \sqrt{8a} \end{array}\right\} \Rightarrow 8=\sqrt{8a} \Rightarrow a=8$$

b) $\displaystyle\int_0^{10} f(x)\,dx = \int_0^8 \sqrt{ax}\,dx + \int_8^{10}\frac{x^2-32}{x-4} = \left(\sqrt{a}\cdot\frac{2}{3}x^{3/2}\right)_0^8 + \int_8^{10}\left(x+4-\frac{16}{x-4}\right)dx =$

$$= \frac{32}{3}\sqrt{2a} + \left(\frac{x^2}{2}+4-16\ln|x-4|\right)_8^{10} = \boxed{\frac{32}{3}\sqrt{2a}+26\ln\frac{2}{3}}$$

(4) $A' = \begin{pmatrix} 1 & 3 & 1 & 5 \\ m & 0 & 2 & 0 \\ 0 & m & -1 & m \end{pmatrix}$; $A = \begin{pmatrix} 1 & 3 & 1 \\ m & 0 & 2 \\ 0 & m & -1 \end{pmatrix}$

a) $|A| = 0 \Rightarrow m^2 + m = 0 \begin{cases} m = 0 \\ m = -1 \end{cases}$

Si $m \neq 0$ y $m \neq -1 \Rightarrow \text{ran}(A) = \text{ran}(A') = n^{o}$ de incógnitas $\Rightarrow$ S.C.D.

$m = 1$: $A' = \begin{pmatrix} 1 & 3 & 1 & 5 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 1 \end{pmatrix}$; $|A| = 2$

Resolvemos por Cramer:

$x = \dfrac{\begin{vmatrix} 5 & 3 & 1 \\ 0 & 0 & 2 \\ 1 & 1 & -1 \end{vmatrix}}{2} = -2$; $y = \dfrac{\begin{vmatrix} 1 & 5 & 1 \\ 1 & 0 & 2 \\ 0 & 1 & -1 \end{vmatrix}}{2} = 2$; $z = \dfrac{\begin{vmatrix} 1 & 3 & 5 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{vmatrix}}{2} = 1$

$\Rightarrow \boxed{x = -2 \; ; y = 2 \; ; z = 1}$

b) $m = 0$:

$\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix} \neq 0 \Rightarrow \text{ran}(A) = 2$

$A' = \begin{pmatrix} 1 & 3 & 1 & 5 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$ filas proporcionales

$\Rightarrow \text{ran}(A') = 2.$

$\text{ran}(A) = \text{ran}(A') < n^{o}$ incógnitas $\Rightarrow$ S.C.I.

$\left.\begin{array}{r} x + 3y + z = 5 \\ -z = 0 \end{array}\right\} \Rightarrow \boxed{\begin{array}{l} z = 0 \\ y = \lambda \\ x = 5 - 3\lambda \end{array}}$

c) $m = -1$:

$\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix} \neq 0 \Rightarrow \text{ran}(A) = 2$

$A' = \begin{pmatrix} 1 & 3 & 1 & 5 \\ -1 & 0 & 2 & 0 \\ 0 & -1 & -1 & -1 \end{pmatrix}$

$\begin{vmatrix} 1 & 3 & 5 \\ -1 & 0 & 0 \\ 0 & -1 & -1 \end{vmatrix} \neq 0 \Rightarrow \text{ran}(A') = 3 \neq \text{ran}(A) \Rightarrow$ S.I.

⑤ $A = \begin{pmatrix} 3 & 1 \\ -2 & -1 \end{pmatrix}$; $B = \begin{pmatrix} 5 & -2 \\ 1 & 3 \end{pmatrix}$; $AXA - B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$\Rightarrow AXA = B \Rightarrow X = A^{-1} B A^{-1}$

A^{-1} : $|A| = -1$

$\left.\begin{array}{l} \\ Adj(A) = \begin{pmatrix} -1 & 2 \\ -1 & 3 \end{pmatrix} \end{array}\right\} A^{-1} = \begin{pmatrix} 1 & 1 \\ -2 & -3 \end{pmatrix}$

$$X = \begin{pmatrix} 1 & 1 \\ -2 & -3 \end{pmatrix} \begin{pmatrix} 5 & -2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -2 & -3 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ -9 & -7 \end{pmatrix}$$

Opción B

1) $\lim_{x\to 0} \frac{x-\alpha \operatorname{sen} x}{x^2} \to$ finito

$$\lim_{x\to 0} \frac{x-\alpha \operatorname{sen} x}{x^2} = \frac{0}{0} \to L'H: \lim_{x\to 0} \frac{1-\alpha\cos x}{2x} = \frac{1-\alpha}{0} = \frac{0}{0} \Rightarrow \alpha = 1$$

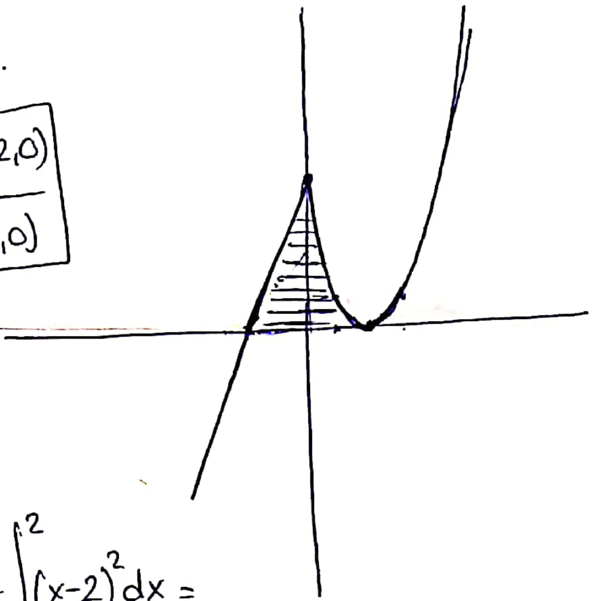
$$L'H: \lim_{x\to 0} \frac{\operatorname{sen} x}{2} = 0$$

2) $f(x) = \begin{cases} 2x+4 \ ; & x \le 0 \\ (x-2)^2 \ ; & x > 0 \end{cases}$

a) Eje de abcisas: $y=0$.

$2x+4=0 \Rightarrow x=-2 \to \boxed{(-2,0)}$

$(x-2)^2=0 \Rightarrow x=2 \to \boxed{(2,0)}$



b) $$\int_{-2}^{2} f(x)\,dx = \int_{-2}^{0} (2x+4)\,dx + \int_{0}^{2} (x-2)^2\,dx =$$

$$= \left(x^2+4x\right)\Big|_{-2}^{0} + \left(\frac{x^3}{3} - 2x^2 + 4x\right)\Big|_{0}^{2} = -4+8+\frac{8}{3}-8+8 = \boxed{\frac{20}{3}\ u^2}$$

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③ $\int_{-1}^{0} \ln(2+x)\,dx = \left(x\ln(2+x)\right)_{-1}^{0} - \int_{-1}^{0} \frac{x}{2+x}\,dx = -\int_{-1}^{0} \frac{x}{2+x}\,dx = -\int_{-1}^{0}\left(1 - \frac{2}{2+x}\right)dx =$

$u = \ln(2+x) \Rightarrow du = \frac{1}{2+x}dx$

$dv = dx \Rightarrow v = x$

$= -\left(x - 2\ln(2+x)\right)_{-1}^{0} = \underline{\underline{2\ln 2 + 1}}$

④ $\left.\begin{array}{l}(b+1)x + y + z = 2 \\ x + (b+1)y + z = 2 \\ x + y + (b+1) = -4\end{array}\right\}$ $A = \begin{pmatrix} b+1 & 1 & 1 \\ 1 & b+1 & 1 \\ 1 & 1 & b+1 \end{pmatrix}$

a) $|A| = (b+1)^3 + 2 - 3(b+1) = b^3 + 3b^2 + 3b + 1 + 2 - 3b - 3 =$

$= b^3 + 3b^2 = 0 \Rightarrow \begin{cases} b = 0 \\ b = -3 \end{cases}$

$\underline{\underline{\text{Si } b \neq 0 \text{ y } b \neq -3}} \Rightarrow \text{Ran}(A) = \text{Ran}(A') = n^{\underline{o}} \text{ incóg} \Rightarrow \underline{\underline{\text{S.C.D.}}}$

Si $\underline{\underline{b=0}}$: $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ $A' = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & -4 \end{array}\right)$ $\begin{vmatrix} 1 & 2 \\ 1 & -4 \end{vmatrix} \neq 0$

$\text{ran}(A) = 1$ $\text{ran}(A') = 2$

$\Rightarrow \underline{\underline{\text{S.I.}}}$

Si $\underline{\underline{b = -3}}$: $A' = \begin{pmatrix} -2 & 1 & 1 & 2 \\ 1 & -2 & 1 & 2 \\ 1 & 1 & -2 & -4 \end{pmatrix} \xrightarrow[-F_1 - F_2 = F_2']{} \begin{pmatrix} -2 & 1 & 1 & 2 \\ 1 & 1 & -2 & -4 \\ 1 & 1 & -2 & -4 \end{pmatrix}$ } iguales

$\begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix} \neq 0 \Rightarrow \text{ran}(A) = 2$ $\Rightarrow \text{ran}(A') = 2$

$\underline{\text{ran}(A) = \text{ran}(A') = 2 < n^{\underline{o}} \text{ incóg.}} \Rightarrow \underline{\underline{\text{S.C.I.}}}$

b) $\underline{b=-3}$: $\left.\begin{array}{l} -2x+y+z=2 \\ x-2y+z=2 \end{array}\right\}$

$\underset{\sim}{z=\lambda} \Rightarrow \left.\begin{array}{l} -2x+y=2-\lambda \\ x-2y=2-\lambda \end{array}\right\}$

$-3y = 6-3\lambda \Rightarrow \underset{\sim}{y=\lambda-2}$

$x = 2-\lambda+2(\lambda-2) = \underset{\sim}{\cancel{6}\lambda-2}$

$$\boxed{\begin{array}{l} x=\lambda-2 \\ y=\lambda-2 \\ z=\lambda \end{array}}$$

5 $A = \begin{pmatrix} 0 & 0 & -1 \\ -1 & 1 & -1 \\ 1 & 0 & b \end{pmatrix}$

a) $A^2-2A+I=0$

$A^2 = \begin{pmatrix} -1 & 0 & -b \\ -2 & 1 & -b \\ b & 0 & b^2-1 \end{pmatrix}$

$A^2-2A+I = \begin{pmatrix} -1-0+1 & 0+0+0 & -b+2+0 \\ -2+2+0 & 1-2+1 & -b+2+0 \\ b-2+0 & 0+0+0 & b^2-1-2b+1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\left.\begin{array}{l} -b+2=0 \\ b^2-2b=0 \end{array}\right\} \left.\begin{array}{l} \Rightarrow b=2 \\ \Rightarrow b=0 \\ \quad\;\; b=2 \end{array}\right\} \Rightarrow \boxed{b=2}$

b) $A = \begin{pmatrix} 0 & 0 & -1 \\ -1 & 1 & -1 \\ 1 & 0 & 0 \end{pmatrix}$; $AX-2A^t=0$

$AX = 2A^t$

$X = 2\cdot A^{-1}\cdot A^t$

$A^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix}$

$A^t = \begin{pmatrix} 0 & -1 & 1 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{pmatrix}$

$X = 2\cdot\begin{pmatrix} -1 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & 1 & -1 \end{pmatrix} = \begin{pmatrix} -2 & -2 & 0 \\ -2 & 2 & -2 \\ 0 & 2 & -2 \end{pmatrix}$