Otras relaciones trigonométricas:

de sumas a productos:

$$\operatorname{sen}\alpha + \operatorname{sen}\beta = 2\operatorname{sen}\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\operatorname{sen}\alpha - \operatorname{sen}\beta = 2\operatorname{sen}\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\operatorname{sen}\frac{\alpha+\beta}{2}\operatorname{sen}\frac{\alpha-\beta}{2}$$

de productos a sumas:

$$\operatorname{sen}\alpha\cos\beta = \frac{1}{2}[\operatorname{sen}(\alpha+\beta) + \operatorname{sen}(\alpha-\beta)]$$

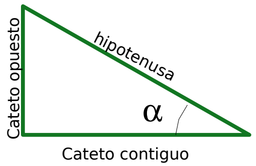
$$\cos\alpha\operatorname{sen}\beta = \frac{1}{2}[\operatorname{sen}(\alpha+\beta) - \operatorname{sen}(\alpha-\beta)]$$

$$\operatorname{sen}\alpha\operatorname{sen}\beta = -\frac{1}{2}[\cos(\alpha+\beta) - \cos(\alpha-\beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$$

TRIGONOMETRÍA

Definiciones:



$$\operatorname{sen}\alpha = \frac{\text{cateto opuesto}}{\text{hipotenusa}}$$

$$\cos\alpha = \frac{\text{cateto contiguo}}{\text{hipotenusa}}$$

$$\operatorname{tg}\alpha = \frac{\text{cateto opuesto}}{\text{cateto contiguo}}$$

Relaciones entre razones trigonométricas:

$$\operatorname{sen}^2\alpha + \cos^2\alpha = 1$$

$$\operatorname{tg}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha}$$

$$\operatorname{cosec}\alpha = \frac{1}{\operatorname{sen}\alpha}$$

$$\sec\alpha = \frac{1}{\cos\alpha}$$

$$\operatorname{cotg}\alpha = \frac{1}{\operatorname{tg}\alpha} = \frac{\cos\alpha}{\operatorname{sen}\alpha}$$

Razones trigonométricas de algunos ángulos:

$$\text{sen } 30^o = \frac{1}{2} \qquad \cos 30^o = \frac{\sqrt{3}}{2} \qquad \text{tg } 30^o = \frac{\sqrt{3}}{3}$$

$$\text{sen } 60^o = \frac{\sqrt{3}}{2} \qquad \cos 60^o = \frac{1}{2} \qquad \text{tg } 60^o = \sqrt{3}$$

$$\text{sen } 45^o = \frac{\sqrt{2}}{2} \qquad \cos 45^o = \frac{\sqrt{2}}{2} \qquad \text{tg } 45^o = 1$$

$$\text{sen } 0^o = 0 \qquad \cos 0^o = 1 \qquad \text{tg } 0^o = 0$$

$\text{sen } 90^o = 1$
$\cos 90^o = 0$
$\text{tg } 90^o \rightarrow$ no existe

$$\text{sen } 180^o = 0 \qquad \cos 180^o = -1 \qquad \text{tg } 180^o = 0$$

$\text{sen } 270^o = -1$
$\cos 270^o = 0$
$\text{tg } 270^o \rightarrow$ no existe

Relaciones trigonométricas con ángulos agudos:

$$\text{sen}(90^o - \alpha) = \cos \alpha \qquad \cos(90^o - \alpha) = \text{sen } \alpha \qquad \text{tg}(90^o - \alpha) = \text{cotg } \alpha$$

$$\text{sen}(180^o - \alpha) = \text{sen } \alpha \qquad \cos(180^o - \alpha) = -\cos \alpha \qquad \text{tg}(180^o - \alpha) = -\text{tg } \alpha$$

$$\text{sen}(180^o + \alpha) = -\text{sen } \alpha \qquad \cos(180^o + \alpha) = -\cos \alpha \qquad \text{tg}(180^o + \alpha) = \text{tg } \alpha$$

$$\text{sen}(-\alpha) = -\text{sen } \alpha \qquad \cos(-\alpha) = \cos \alpha \qquad \text{tg}(-\alpha) = -\text{tg } \alpha$$

Teorema del seno:

$$\frac{\text{sen } \alpha}{a} = \frac{\text{sen } \beta}{b} = \frac{\text{sen } \gamma}{c}$$

Teorema del coseno:

$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos \alpha$$

$$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos \beta$$

$$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos \gamma$$

Razones trigonométricas del ángulo suma:

$$\text{sen}(\alpha + \beta) = \text{sen } \alpha \cos \beta + \cos \alpha \text{ sen } \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \text{sen } \alpha \text{ sen } \beta$$

$$\text{tg}(\alpha + \beta) = \frac{\text{tg } \alpha + \text{tg } \beta}{1 - \text{tg } \alpha \text{ tg } \beta}$$

Razones trigonométricas del ángulo diferencia:

$$\text{sen}(\alpha - \beta) = \text{sen } \alpha \cos \beta - \cos \alpha \text{ sen } \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \text{sen } \alpha \text{ sen } \beta$$

$$\text{tg}(\alpha - \beta) = \frac{\text{tg } \alpha - \text{tg } \beta}{1 + \text{tg } \alpha \text{ tg } \beta}$$

Razones trigonométricas del ángulo doble:

$$\text{sen}(2\alpha) = 2 \text{ sen } \alpha \cos \alpha$$

$$\cos(2\alpha) = \cos^2 \alpha - \text{sen}^2 \alpha$$

$$\text{tg}(2\alpha) = \frac{2 \text{ tg } \alpha}{1 - \text{tg}^2 \alpha}$$

Razones trigonométricas del ángulo mitad:

$$\text{sen}\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$\text{tg}\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$$